The hybrid vertical representation

The vertical coordinate in global models is usually defined using the *hybrid vertical representation*. In such a system the vertical layers are defined by their pressure:

$$p_k = A_k + B_k p_{\text{surf}} \tag{1}$$

for $k = 1, \ldots, N$. The coefficients A_k and B_k are constants whose values effectively define the vertical coordinate, and p_{surf} is the surface pressure.
In the EMAC model the hybrid coefficients are called `hyam` and `hybm` for the level *midpoint* (i.e. the *center* of the cell), and `hyai` and `hybi` for the level *interfaces* (i.e. the *borders* of the cell). The surface pressure is indicated by `aps`. In the specific example considered here, there are 19 vertical layers, i.e. the dimension of `hyam` and `hybm` is 19, whereas the dimension of `hyai` and `hybi` is 20 (see Figure 1).

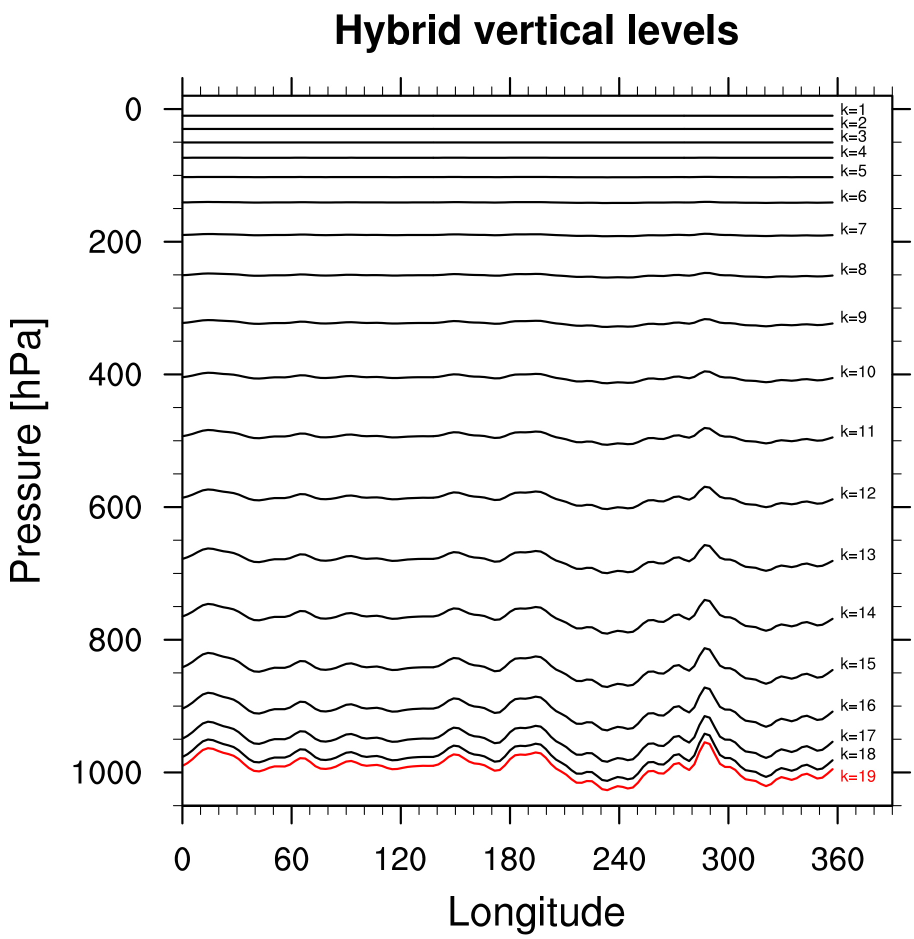

Figure 1: Vertical hybrid levels from the EMAC model in the T42L19 configuration. The plot shows a selected slice at latitude $\simeq 52°$S.

Exercise 1

- download the sample data file at http://www.pa.op.dlr.de/~MattiaRighi/NCL/
- open the sample data file `NetCDF_sample.nc` and read the variable `O3`, representing ozone concentration in December 2004.
- what are the units of `O3`? Convert them to ppbv.
- interpolate this variable from hybrid to pressure levels, using 50 equally-spaced pressure levels ranging from 100 to 1000 hPa (**HINT:** use the NCL function `vinth2p`).
- extract the timestep corresponding to 15.12.2004 at 12:00 (**HINT:** use the NCL function `ut_inv_calendar` and coordinate subscripting).
- compute the zonal average (average of the longitude coordinate).
- plot the result as a pressure-height/latitude plot (**HINT:** check the examples gallery on the website).
- customize the plot to get something similar to Figure 2 (**HINT:** use the `wh-bl-gr-ye-re` color map).

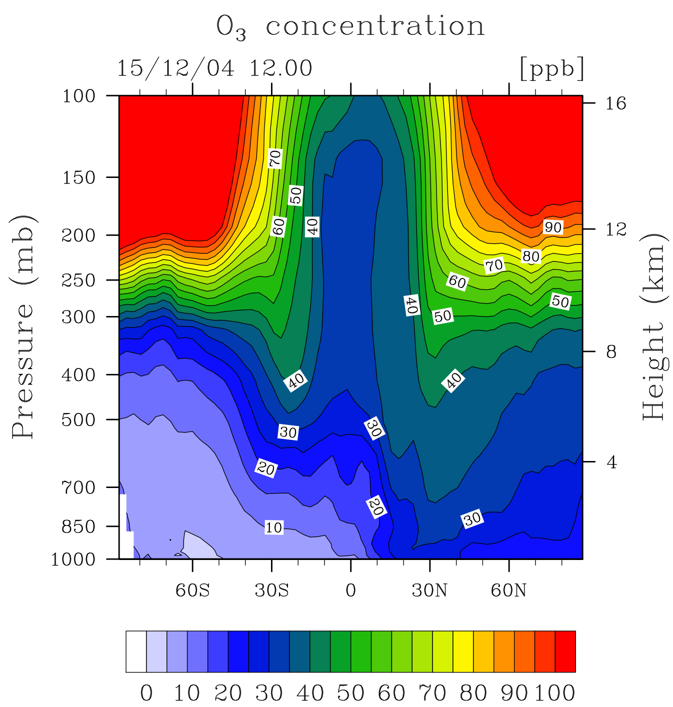

Figure 2: Plot of the exercise 1

Exercise 2

- open the sample data file and read the variable `CO` at the lowermost model level (levels are ordered top-to-bottom).
- convert units to ppbv.
- average the variable over the longitude range [150°E,110°W] (**HINT:** use coordinate subscripting).
- extract the values at the latitudes corresponding to indexes 10, 30 and 50. This will result in an array of dimensions `(ntime, 3)`.
- plot this variable using a multiple XY plot.
- customize the plot using three different colors, a single dash pattern and a value of 2 for the line thickness.
- write the units on the Y axis, add three text labels as a legend and a title specifying the longitude range used in the average, as in Figure 3 (**HINT:** use the function `gsn_add_text` or `gsn_text_ndc`).

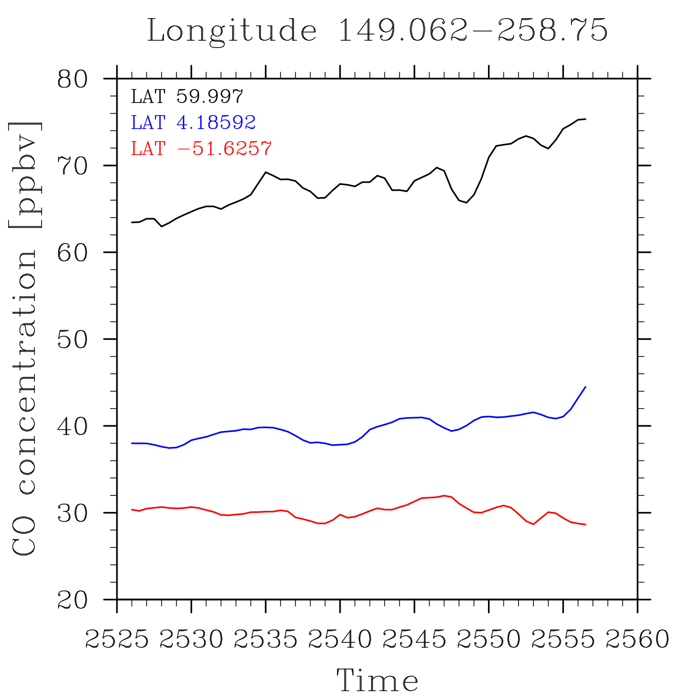

Figure 3: Plot of the exercise 2

The logistic equation

The logistic equation is a simple way to understand the concept of chaos. It was originally introduced to approximate the evolution of an animal population over time. Consider a population that has a generation per year. If x_n is the number of animals in the year n, then the number of animal in the following year $n+1$ is written as:

$$x_{n+1} = r\, x_n\, (1 - x_n), \tag{2}$$

where r is the growth rate (or fecundity) of the population and the factor $(1 - x_n)$ accounts for the carrying-capacity of the environment. This means that the population cannot grow beyond a certain limit (for example, because of limited resources).
For a given initial seed value x_0, one can show that depending on the value of the growth rate r the evolution of the population follows different behaviours: *fixed* (the population approaches a stable value), *periodic* (the population oscillates between two or powers of two fixed values) or *chaotic* (the population assumes an infinite number of values in the range [0,1]). This can be visualized with a bifurfaction diagram (Figure 4), where the values assumed by the population are plotted as a function of r.

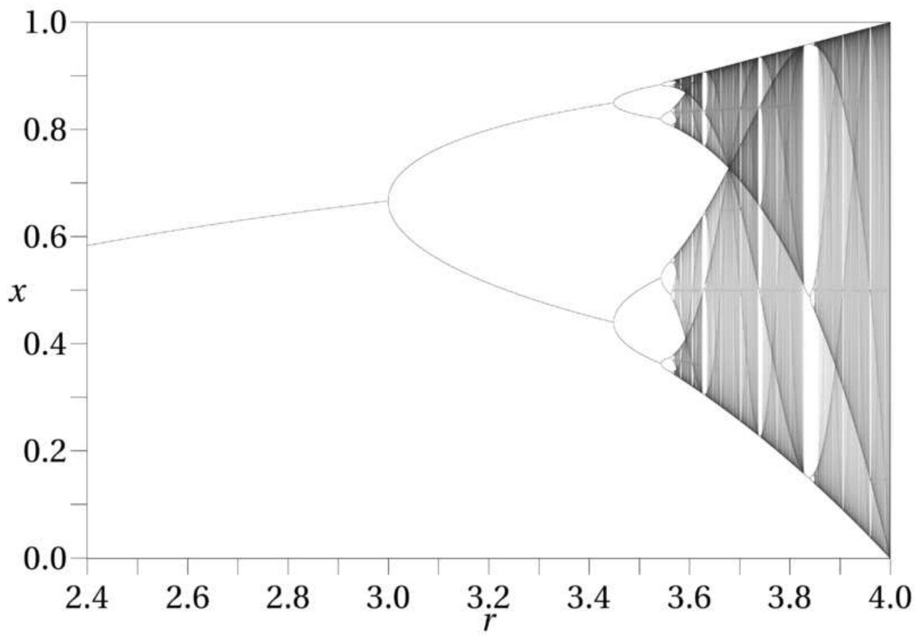

Figure 4: Bifurcation diagram for the logistic equation (from Wikipedia).

Exercise 3

Apply the logistic equation described in the previous page to obtain a bifurcation diagram similar to the one in Figure 4.

- use $x_0 = 0.5$ as initial seed.
- apply the equation for 300 values of r in the range [1;4].
- for each value of r consider $n = 300$ iterations.
- skip the first 100 iterations when plotting the population values.
- plot the values of x as a scatter plot (check the corresponding example on the website), with r on the horizontal axis (**HINT:** the array to be plotted is a function of r and of the iteration index (two-dimensional). It needs to be converted to one-dimensional before plotting).

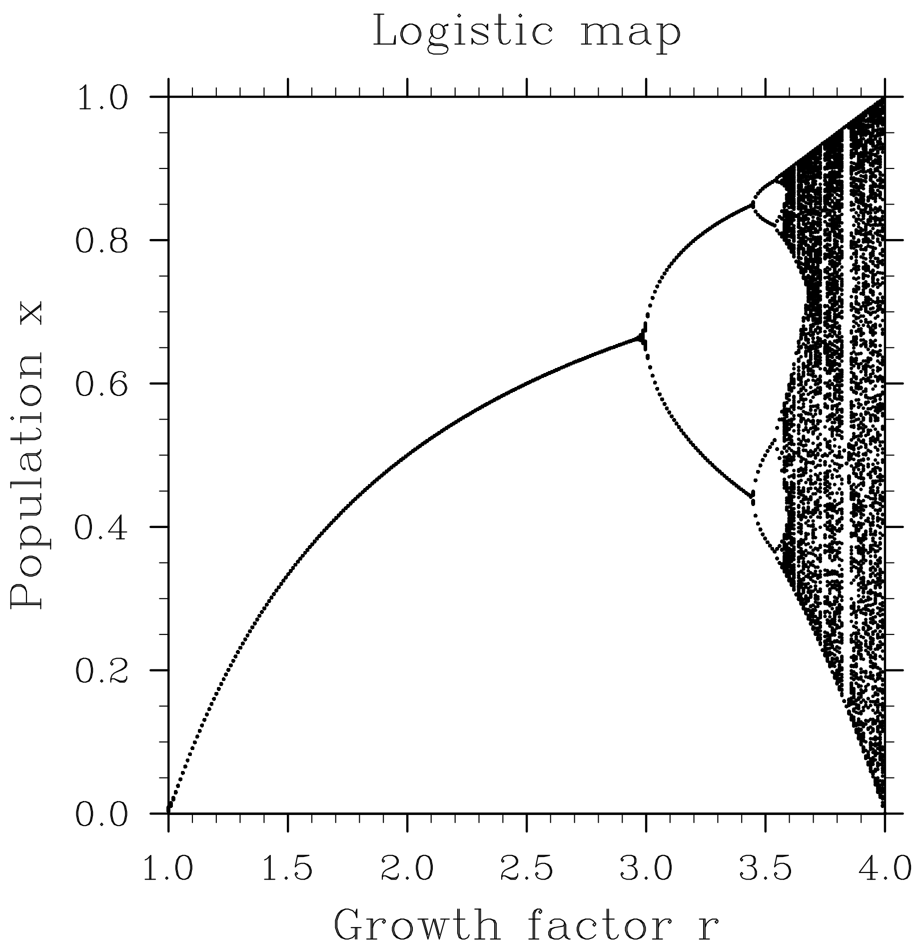

Figure 5: Plot of the exercise 3