Lecture, Summer term, LMU München

An Introduction to Atmospheric Modelling

Lecture 7: Dynamics of the Atmosphere

Content:

- Scales of Atmospheric Motion
- Local Winds
- Global Circulation
- Numerical Description in a Global Circulation Model

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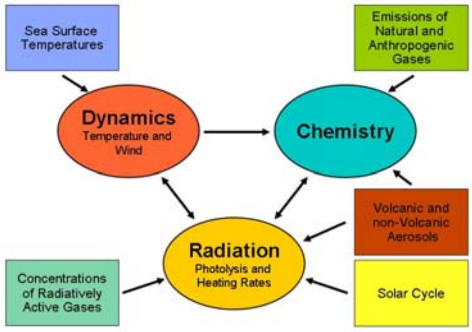
Components of a coupled chemistry-climate model (CCM)



Principle Forces and Definitions

Principle Forces and Definitions

Principle forces and Definitions

Q: What drives the weather in the atmosphere?
Q: What causes the weather to change?

We must analyse the predominant forces in the atmosphere, namely:

- Pressure and pressure gradients
- Gravity
- Rotation of the earth
- Friction

Principle Laws and Approximations

Principle Laws and Approximations

Principle Laws and Approximations (Atmospheric Motion)

Atmospheric motions are governed by three fundamental physical principles:

- **Newton's second law** (Conservation of momentum)
- **Continuity Equation** (Conservation of Mass)
  - relates convergence of horizontal winds with vertical motion
- **Thermodynamic Equation** (1st Law of TD, Conservation of Energy)
  - balance of heating & heat transport
- **Equation of State**
  - relates pressure, density and temperature
- **Hydrostatic Approximation**
  - pressure force versus gravity

Newtons' Second Law

F = m a

The <u>acceleration</u> of an object is directly proportional to the <u>net force</u> acting on the object…
…and inversely proportional to the <u>mass</u> of the object.

Conservation of momentum

$$(1)\quad \frac{D\vec{v}_H}{Dt} = -f\,k\times\vec{v}_H - \nabla_p\Phi + \vec{F}_H$$

Balanced Forces:

- Pressure Gradient Force
- Coriolis Force
- Friction

Frictional Force

Pressure Gradient

Coriolis Force

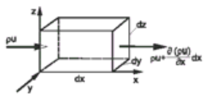
Continuity Equation

mathematically:

$$\boxed{\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0}$$

where ρ is the density of dry air,and **V** is the three-dimensional velocity vector.

means: overall mass transport balances (in = out)



Thermodynamic equation

Conversation of energy: dQ = dU + pdV dQ, dU, dV are increments in total energy, internal energy and volume

$$\boxed{\frac{\partial T}{\partial t} = -\vec{v}\cdot grad\ \ T + \omega\cdot\left(\kappa\frac{T}{p} - \frac{\partial T}{\partial p}\right) + Q/c_p}$$

Local temporal change in temperature | Horizonal advection of heat | Vertical Transport | Diabatic heating or cooling

Means: "continuity equation" for heat:

Local temporal change in temperature
= horizontal advection of heat
+ vertical transport
+ diabatic heating or cooling (radiation, latent etc.)

The Equation of State

- **Ideal Gas**
  - An ideal gas can be characterized by **three state variables**: absolute pressure (P), volume (V), and absolute temperature (T). The relationship between them may be deduced from kinetic theory
  - Air is an ideal gas, to good approximation: the pressure, density and temperature are related by the ideal gas law can be thought of either as a wave or as a particle that represents the movement of energy through space

Ideal gas law: $PV = nRT = NkT$

- n = number of <u>moles</u>
- R = universal gas constant = 8.3145 J/mol K
- N = number of molecules
- k = Boltzmann constant = 1.38066 x 10^{-23} J/K = 8.617385 x 10^{-5} eV/K
- k = R/N_A
- N_A = Avogadro's number = 6.0221 x 10^{23}

Hydrostatic Approximation

Scale Analysis for vertical component of the momentum equation:

Difference in pressure is balanced by gravity:
good approximation for the vertical dependence of the pressure field in the real atmosphere

$$\boxed{dp = -\rho g dz}$$

<u>Approximation:</u>

Pressure varies with height as if the air was motionless

<u>Features:</u>

ρ and g are both positive => p decreases with height

Thickness of a layer between p1 and p2 is proportional to the average temperature

$$\frac{dp}{dz} = -\frac{pg}{RT} \quad => \quad \int_{p_0}^{p_1}\frac{RT}{g}\,d\log p = \underbrace{\int_{p_0}^{p_1}dz}_{} = z_1 - z_0$$

Thickness of a layer

Scale Analysis

$$\mathbf{u} = \mathbf{U} + \mathbf{u}'$$

Where **U** represents the large scale motion and **u'** the small scale motion

Navier-Stokes Equations

- A set of **nonlinear partial differential equations** that describe the **flow of fluids such as liquids and gases.**
- For example: they **govern the movement of air in the atmosphere, ocean currents, water flow in a pipe, as well as many other fluid flow phenomena.**
- The equations are derived by considering the **mass, momentum and energy balances** for an infinitesimal control volume.
- The Navier-Stokes equations need to be augmented by an **equation of state** for compressible flows. The variables to be solved for are the **velocity components, the fluid density, static pressure and temperature.**
- Solution depends on the **fluid properties such as viscosity, specific heats, thermal conductivity etc** and on the boundary conditions of the domain of study.

$$\nabla \cdot \vec{u} = 0$$

$$\frac{d\vec{u}}{dt} = -(\vec{u} \cdot \nabla)\vec{u} - \frac{1}{\rho}\nabla p + \nu\nabla^2\vec{u} + \vec{f}$$

convection pressure viscosity external forces

Representation of Dynamical Processes in Global Atmospheric Models

- **Physical Basis of Climate Modelling:**
  - mathematical representation of the physical laws that govern the climate behaviour
  - from such model's the future behaviour or future climate may be determined
- **Atmospheric primitive equations**
  - A set of governing equations that describe large-scale atmospheric motions
  - can be derived from conservation laws governing momentum, mass, energy, and moisture (see Holton, 1979 - Chap. 2).
  - physics of the various interactive processes, serving to link the processes together
  - These are called the primitive equations, not because they are crude or simplistic but because they are fundamental or basic. Using the Eulerian framework in x-y-p coordinates, they can be written as follows:
  - in case of the atmosphere these laws are expressed by the equations which describe the change of momentum, temperature, and moisture

Atmospheric Primitive Equations

- **Atmospheric primitve equations**
  - The governing dynamic, thermodynamic, and conservation equations are mapped to a spherical geometry
  - reducing set by the following
  - Basic prediction variables:
    - discretized horizontal wind components (u and v)
    - Temperature
    - water vapour represented by specific humidity (q)
    - surface pressure

Dynamics

Prognostic variables: vorticity, divergence, temperature, specific humidity, log-surface pressure, cloud water, optional 21 Tracer
Surface boundary conditions: SST, orography, land-sea mask, albedo, roughness length, vegetation
Physical Parameterizations: radiation, clouds, precipitation, convection, PBL, land-surface processes, horizontal diffusion, GWD

Atmospheric Primitive Equations

(1) $\frac{D\vec{v}_H}{Dt} + f\,k \times \vec{v}_H + \nabla_p \Phi = \vec{F}_H$ (2) $\frac{Dc_p T}{Dt} + \omega\alpha = Q$ (3) $\frac{Dq}{Dt} = S$

$$\vec{v}_H = (u, v, 0)$$

Horizontal velocity vector: First prognostic variable

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$$

Properties of a change with time and location a=a(t,x,y,z)

D/Dt denotes the rate of change moving with a fluid particle

(1) from conservations of energy

Atmospheric Primitive Equations

(1) $\frac{D\vec{v}_H}{Dt} + f\,k \times \vec{v}_H + \nabla_p \Phi = \vec{F}_H$ (2) $\frac{Dc_p T}{Dt} + \omega\alpha = Q$ (3) $\frac{Dq}{Dt} = S$

Change of horizontal velocity vector: determined by

Frictional Force

Pressure Gradient

Coriolis Force

Atmospheric Primitive Equations

$$(1)\quad \frac{D\vec{v}_H}{Dt} + f\,k\times\vec{v}_H + \nabla_p\Phi = \vec{F}_H \qquad (2)\quad \frac{Dc_pT}{Dt} + \omega\alpha = Q \qquad (3)\quad \frac{Dq}{Dt} = S$$

$\omega = \frac{Dp}{Dt}$ α : specific volume

c_p : specific heat at constant pressure

Temperature: Second prognostic variable

Based on the first law of thermodynamics, Equation (2) is the thermodynamic equation:

Local temporal change in temperature
= horizontal advection of heat
+ vertical transport
+ diabatic heating or cooling (radiation, latent etc.)

(2) from conservation of energy

Atmospheric Primitive Equations

$$(1)\quad \frac{D\vec{v}_H}{Dt} + f\,k\times\vec{v}_H + \nabla_p\Phi = \vec{F}_H \qquad (2)\quad \frac{Dc_pT}{Dt} + \frac{Dp}{dt}\alpha = Q \qquad (3)\quad \frac{Dq}{Dt} = S$$

Water vapour represented by specific humidity q: Third prognostic variable

Change in the rate of moisture

(3) from conservation of moisture

Atmospheric Primitive Equations

$$(1)\quad \frac{D\vec{v}_H}{Dt} + f\,k\times\vec{v}_H + \nabla_p\Phi = \vec{F}_H \qquad (2)\quad \frac{Dc_pT}{Dt} + \frac{Dp}{Dt}\alpha = Q \qquad (3)\quad \frac{Dq}{Dt} = S$$

Momentum Equation | **Thermodynamic Equation** | **Moisture conservation**

Eq. 1-3 involve six variables $\vec{v}_H,\ T,\ q,\ \omega,\ \alpha,\ \Phi$

It is necessary to have three additional independent equations in order to formally complete the dynamical system.

These equations are provided by the **equation of mass continuity (4)**, the **hydrostatic approximation (5)**, and the **equation of state (6)**

Those equation describe the balances or equilibria which must exist among the variables at all times

Serve to determine the distributions of ω, Φ and α in terms of prognostic variables

$$(4)\quad \frac{Dp_s}{Dt} + p_s(\nabla_p\bullet\vec{v}_H + \frac{\partial\dot{p}}{\partial p}) = 0 \qquad (5)\quad \frac{\delta\Phi}{\delta p} + \alpha = 0 \qquad (6)\quad p\alpha - RT = 0$$

Mass conservation | **Hydrostatic approximation** | **Equation of state**

Atmospheric Primitive Equations: Prognostic Variables

Basic Prognostic Variables in a GCM

Discretized horizontal wind components u and v
Temperature T
Water vapour represented by specific humidity q
Surface pressure

Furthermore models generally include
+ predictive equations for several surface fields
+ predictive equations for liquid water
+ predictive equations for ozone fields etc.

Atmospheric Primitive Equations: Predictive Equation

Example for a predictive Equation for surface pressure

$$(4)\quad \frac{Dp_s}{Dt} + p_s(\nabla_p\bullet\vec{v}_H + \frac{\partial\dot{\sigma}}{\partial\sigma}) = 0$$

Mass conservation

$$\frac{\partial p_s}{\partial t} = -\int_0^1 \nabla_p \bullet (p_s\vec{v}_H)\,d\sigma$$

$$\sigma = \frac{p}{p_s}$$

$$\omega = \frac{Dp}{Dt} = \sigma\vec{v}_H\,\nabla p_s - \int_0^1 \nabla_p \bullet (p_s\vec{v}_H)\,d\sigma$$

Prognostic Variables in ECHAM

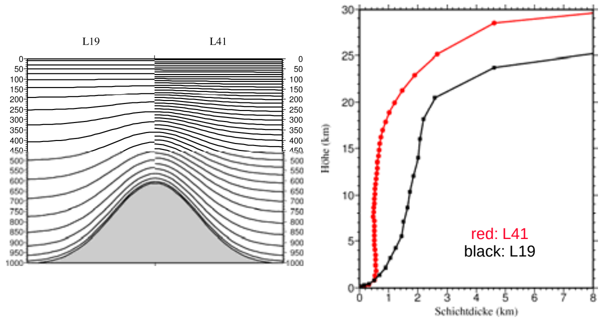
Basic Prognostic Variables in the GCM ECHAM

Vorticity
Divergence
Temperature T
Water vapour represented by specific humidity q
Log-surface pressure
Cloud water content

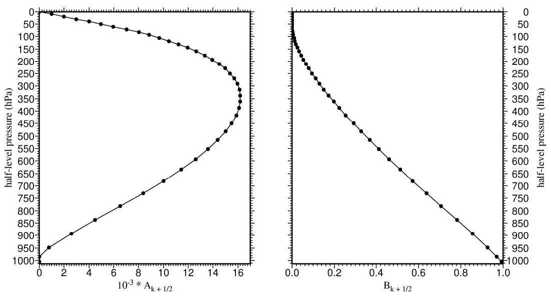
Vertical Discretization

- **Define Vertical Coordinate**
  - The vertical structure of model variables is most commonly represented by values defined at a number of levels in the vertical.
  - For the usual sigma coordinate, the pressure of a certain level is proportional to the surface pressure
  - Therefore coordinate surfaces rise over rather than intersect mountains
  - All prognostic variables are defined at the same level

Vertical Discretization in ECHAM5.L41



ECHAM5.L41 vertical coordinate coefficients



$$p_{k+1/2} = A_{k+1/2} + B_{k+1/2} \cdot p_s$$

Horizontal Discretization: Finite Difference Method

- **Finite-Difference Method:**
  - Variables are represented at one or a variety of grids, which vary according to the relative locations of wind and temperature location
  - According to whether the grid separation is regular in longitude or physical distances as the poles are approached
  - Truncation Error: The residual, when the grid-point values of the **true solution** are substituted into the difference equation
  - **small-scale motions must be truncated due to horizontal and vertical discrestization**
  - The effects of the **unresolved-scale variables on the resolved scales must be treated**, giving an apparent friction F and apparent heating Q. Similar S becomes an apparent source.

Horizontal Discretization: Spectral Method

- **Spectral Method:**
  - Predictive Variables, generally include vorticity and divergence rather than the horizontal wind components
  - **Predictive Variables are represented in terms of truncated expansions of spherical harmonics**
  - Nonlinear terms are evaluated on an almost regular latitude-longitude grid, and the spectral tendencies calclulated by quadrature
  - Triangular Truncation (N=M) most common
  - More about Spectral Models in Lecture 8 General Circulation Models (GCMs)

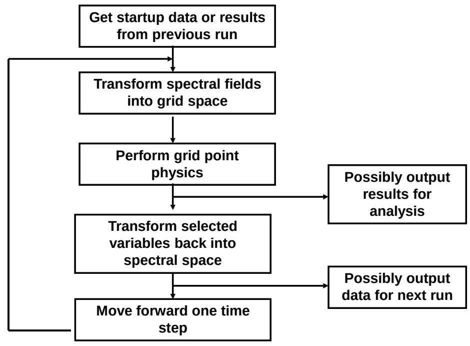
$$X(\lambda,\phi,\sigma,t) = \sum_{m=-M}^{M} \sum_{n=|m|}^{N} X_n^m(\sigma,t)\, P_n^m(\sin\phi)\, e^{im\lambda}$$

X	Any dependent variable
ϕ	Latitude
P_n^m	Associated Legendre functions

Why spectral models?

- Many wave-like features of the atmosphere are best simulated with wave formulation
- However, not usually in all directions
- a rectangular grid is used for vertical transfers, and radiative transfer and surface processes are modeled in this grid space
- the data fields are transformed to grid space at every time step via fast Fourier transforms and Gaussian quadrature (a form of numerical integration) and back to spectral space via Legendre and Fourier transforms.
- time stepping is performed with the waveform representation and grid-point physics is incorporated after transformation into grid space.
- Roughly physics means everything that is not described in the primitive equations.

Flow through the computational scheme of a spectral AGCM



End of Lecture 6