

# Inter-radar comparison accounting for partially overlapping volumes

Zlatko R. Vukovic<sup>1</sup>, Jim M.C. Young<sup>1</sup>, and Norman Donaldson<sup>2</sup>

<sup>1</sup>*Meteorological Service of Canada, Environment Canada, 4905 Dufferin Street, Toronto, Canada*

<sup>2</sup>*Science and Technology Branch, Environment Canada, 4905 Dufferin Street, Toronto, Canada*

(Dated: 18 July 2014)



Zlatko Vukovic

## 1 Introduction

The Meteorological Service of Canada (MSC) of Environment Canada (EC) requires a reliable, accurate and automated radar network performance monitoring tool of inter-radar consistency. This requirement has increased priority due to recently announced funding for improvements to the radar network. Given the current reliance on the Z-R relationship to retrieve precipitation estimates from observed reflectivity, it is essential that the scanning radar be well calibrated. The calibration of the individual radars has recently been shown to vary as much as a few dBZ (Williams, 2005). Such differences can lead to substantial errors in precipitation estimates (Ulbrich and Lee 1999). Consistent calibration between adjacent network radars is also crucial for constructing radar mosaics used in real-time weather forecasting. There are many techniques in use today for verification of radar calibrations, including routine engineering measurements of the various subsystems and a very different approach that combines the radar calibration Z-R relationships through the comparison of radar-derived rain maps with dense networks of rain gauges. Absolute calibration of meteorological radars is still a critical task for meteorological applications. A number of calibration techniques based on external sources were reviewed by Atlas (2002). Current implementations utilize simplified rules for sampling and comparing operational data.

Observing the MSC constraint that only data available from the operational volume scans should be used, a new technique to monitor radar calibration has been developed at Environment Canada. The main objective of this paper is to show a new methodology for evaluating comparable neighboring radar data that will improve the accuracy of inter-radar comparison. An ideal inter-radar comparison between adjacent radars would be the synchronized measurement of the same equidistant volume space and for real world comparisons one needs to assess the degree to which this is achieved. To achieve this goal we outline how a geometric set of points at equal distance from both radars can be accurately derived and create a common inter-radar space (CIS) and extend that idea to allow for partially overlapping volumes. The intersection of the volumes produces a number of CIS points and an operational inter-radar comparison tool has been developed that accounts for the volume of CIS points as a weighting factor in the new Z algorithm comparison (ZAC) methodology. We present the benefit of the ZAC methodology with a case study.

## 2 The ZAC methodology

The idealized procedure for inter-radar comparison relies on the time-space synchronization measurements of a target at equal distance. Namely, three main assumptions are:

1. the target is at equal distance from both adjacent radars;
2. occupying the same volume of the atmosphere; and
3. measurements from both radars are at the same time.

In an operational implementation the above assumption will never be completely satisfied. The next three subsections discuss how achievable the three assumptions are, and how they can be quantified. The fourth subsection, elaborates how to process the collected data for radar-radar comparison. The objective is to establish where overlapping volumes should be found and then to establish the degree to which real operational data meet the overlap criteria. Data with better overlap can be given higher weight in radar-radar comparison.

### 2.1 The Common Inter-radar Space

This section gives an overview of the geometry of a single radar target point as seen by two radars. More detail is given in (Vukovic et al. 2013). A geocentric coordinate system (GCS), in which the Earth is modeled as an oblate spheroid, was chosen as the prime framework with the coordinate origin at the center of the Earth, Figure 1.

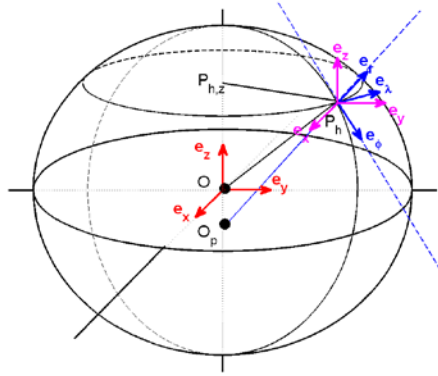


Figure 1: A geocentric coordinate system (GCS) of reference, red, and a local coordinate system, blue.

The position vector of a common target point of two radars relative to the GCS, Figure 2, is the composition of the three vectors: a position vector from the geocentric origin to the surface of the oblate geoid at the geographical latitude and longitude of a radar location, an altitude vector that is normal to the oblate spheroid surface at the point of the radar location and which represents the height of the radar antenna above mean sea level, and at third vector is a local position vector of a common target point measured from the radar. Similarly, the location of the same target point from the second radar in reference to the GCS is given as a composition of the corresponding three vectors.

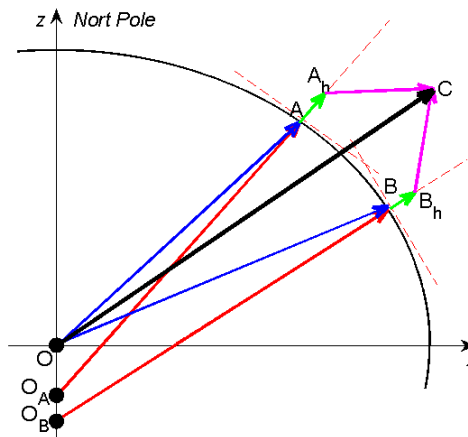


Figure 2: The geometric construction of the geocentric position vector of point C (bold black line) from the geodetic (red color), geocentric (blue color), and local point vectors (pink color) from two locations and altitudes (green color).

Since the two sets of three vectors describe the position of same target with respect to the Earth centre, equating them results in three scalar equations. We assume that the radar antenna locations are known and the azimuth and elevation are specified from the first radar. Using the three scalar equations it was possible to obtain the three remaining unknowns: local azimuth and elevation from the second radar as well as the equal distance. As example of the CIS calculation, the coordinates of two EC radars CWKR (lat = 43.96°, lon = -79.57°, alt = 360 m) and CWSO (lat = 43.37°, lon = -81.38°, alt = 303 m) were used for two different resolutions (1° and 0.1°) of the independent variables, azimuth and elevation, of one of adjacent radars (WKR). Figure 3 shows the CIS points for 1° (a) and 0.1° (b) angle resolution. As expected, the equal-distance is symmetrical in regards to the vertical plane in WKR-WSO direction and the number of equidistant points decreases with the elevation angle. The shape of the CIS doesn't change with angle resolutions, only the density of points is changing.

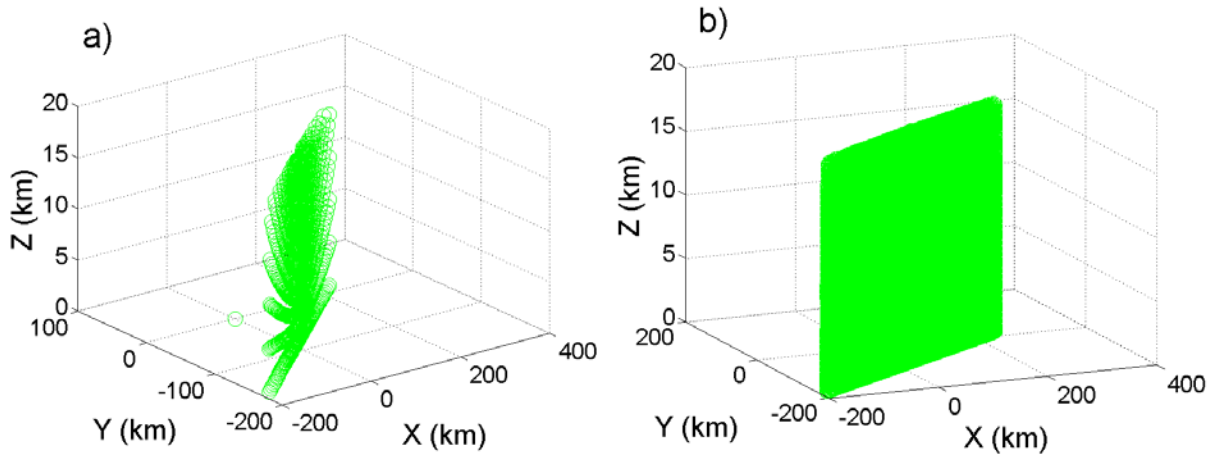


Figure 3: Graphical presentation of calculated CIS points for the WKR (King City, lat = 43.96°, lon = -79.57°, alt = 360 m) and WSO (Exeter, lat = 43.37°, lon = -81.38°, alt = 303 m) pair of radars in the local WKR radar Descartes reference system for azimuth angle resolutions 1° (a) and 0.1° (b).

## 2.2 The overlap coefficients

In reality the assumption of comparing radar reflectivity from the same volume of the atmosphere is almost never satisfied. Figure 4 shows a sketch of the cross section of the overlapping volume  $v_{AB}$  (green) of beam sample volumes of neighbour radars A ( $v_A$ , blue) and B ( $v_B$ , red).

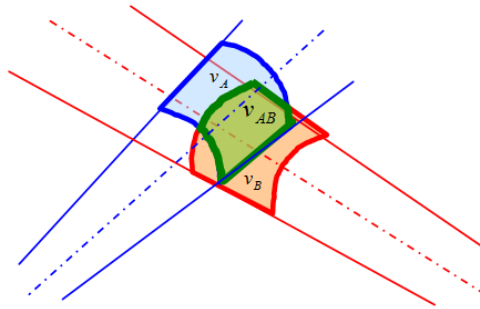


Figure 4: A cross section of overlapping volume  $v_{AB}$  (green) of a beam sample volumes of neighbour radars A ( $v_A$ , blue) and B ( $v_B$ , red).

To quantify the degree to which the same volume is sampled from both radars we will define an overlap coefficient  $\psi$  as a ratio of the overlapping beam volume  $v_{AB}$  and a geometric mean of sample volumes  $v_A$  and  $v_B$ :

$$\psi(v_A, v_B) = \frac{v_{AB}}{\sqrt{v_A v_B}} \quad (2.1)$$

If the overlapping coefficient is close to one, the two radar beams are close to sampling a same target volume. Conversely if the overlapping coefficient is smaller there are more differences between the same sampling volumes. We can interpret the overlap coefficient  $\psi$  as a weight factor that discriminates how well the measurements from the two radar beams are aligned.

For most meteorological radars the fundamental unit of data output from the signal processor is a ray of data. Each ray consists of the integration of the desired number of pulses (samples), collected while the antenna is rotating. . Therefore our compared volumes are compositions of multiple numbers ( $N_A$  and  $N_B$ ) of individual pulse volumes. Consequently, the equation (2.1) for the overlapping coefficients is modified:

$$\psi(v_A, v_B) = \frac{V_{AB}}{\sqrt{v_A v_B N_A N_B}} \quad (2.2)$$

Where  $V_{AB}$  is a sum of all overlaps between individual pulses.

Since the geometry of the individual pulse could be well approximated as a segment of the cone, it is easy to calculate it.

$$v = \pi w r^2 \left[ \sin^2 \gamma - \left( 1 + \frac{w^2}{12r^2} \right) (1 - \cos \gamma)^2 (1 + 2 \cos \gamma) \right] \quad (2.3)$$

Where  $w$  is a beam length;  $r$  is radial CIS distance; and  $\gamma$  is beam width in degrees.

Finding an analytical solution for the overlap volume is very complex task and therefore a numerical approach was taken. Using an appropriate accuracy of the numerical estimation of the overlap volume together with Equation (2.3), for known number of samples per radial resolution, we are able to obtain the overlap coefficients. i.e. the weight factors, for each CIS point. The calculated weigh factors quantities the quality of CIS points for radar-radar comparison in a particular CIS point.

### 2.3 The time synchronization

Two neighbouring radars might have not only different antenna speeds, but also different scanning schemes which cause a lack of synchronization in measurements of same target. If measurements from radar A and B occurred in moments  $t_A$  and  $t_B$ , respectfully, then the time difference is:

$$\Delta t = |t_A - t_B| \quad (2.4)$$

Comparison of reflectivity measured in closer time, smaller  $\Delta t$ , means measuring radar reflectivity from the same volume target with more equal content and atmospheric conditions. And reverse, if  $\Delta t$  is larger there is more chance that content and atmospheric conditions in the target volume of two radar's measurements were significantly changed.

Similar to the spatial synchronisation, we can define a weighting factor to reflect the degree of time synchronisation. There are many ways to parameterize the time synchronisation weight factor that satisfies condition that it is equal one for  $\Delta t = 0$  and monotonically decreases as  $\Delta t$  increase. If we chose exponential function:

$$\xi = e^{-\frac{\Delta t}{T}} \quad (2.5)$$

then selection of constant  $T$  will determine how fast the reduction is.

For different weather conditions and atmospheric processes the constant  $T$  will be different. For summer, when the atmospheric dynamics are much more intense than during winter, we could expect smaller  $T$  values and more influence of  $\Delta t$  on the synchronization factor  $\xi$ . From that reasons it is more reasonable to treat  $T$  as a parameter that depends of weather conditions. As a first approximation for  $T$  parameter we could use inverse average relative variation of reflectivity with time:

$$T = \frac{1}{\frac{1}{Z} \frac{\delta Z}{\delta t}} = \frac{\overline{\delta t}}{\overline{\delta Z}} \quad (2.5)$$

Where  $\overline{Z}$  and  $\frac{\overline{\delta Z}}{\overline{\delta t}}$  are the average logarithmic radar reflectivity factor and average change the logarithmic radar reflectivity factor obtained from both radars in the CIS points during an appropriate time period.

### 2.4 Weighted orthogonal regression

The comparison of two neighboring radars has two sets of data that were obtained in time-space as close as were operationally possible at equal distance from both radars. Since both measurements of radar reflectivity,  $Z_{A,n}$  and  $Z_{B,n}$ , are independent, with similar errors, the most appropriate method to find a best linear regression line between them is to use the orthogonal regression line rather than the common least squares regression line. The orthogonal regression line minimizes the sum of the squared perpendicular distances from the each observation to the regression line. In addition to that, for simple statistical regression analysis it is assumed that all data pairs are equally significant, which is not the case here because of the space and time synchronisation weight factors  $w$  and  $\xi$ . A pair of reflectivity  $Z_A$  and  $Z_B$  that has a larger beam overlap and better coincidence time is much more significant for comparison than a pair of reflectivities where the beams barely overlap and the measurements were done at decidedly different times. Therefore we will include the weights  $w_n$  and we will obtain the regression line:

$$\hat{Z}_A = \kappa \hat{Z}_B + Z_o \quad (2.6)$$

such that the weighted sum of squared residuals of the model is minimized.

$$D^2 = \sum_{n=1}^N w_n \left[ \left( \hat{Z}_{A,n} - Z_{A,n} \right)^2 + \left( \hat{Z}_{B,n} - Z_{B,n} \right)^2 \right] \quad (2.7)$$

where  $(\hat{Z}_{B,n}, \hat{Z}_{A,n})$  is the point on the regression line  $\hat{Z}_A = \kappa \hat{Z}_B + Z_o$  that is closest to measured reflectivity  $(Z_{B,n}, Z_{A,n})$ . The weight factor  $w_n$  is defined as products of the spatial ( $\psi_n$ ) and time ( $\xi_n$ ) weights,  $w_n = \psi_n \xi_n$ . The total number of the CIS points where reflectivity was compared is denoted as N. Minimizing  $D^2$  with respect to  $\kappa$  and  $Z_o$  gives equations of the form:

$$a\kappa^4 + b\kappa^3 + c\kappa^2 + dk + e = 0 \quad (2.8)$$

$$Z_o = \bar{Z}_A - \kappa \bar{Z}_B \quad (2.9)$$

Equation (2.8) has 4 solutions, two imaginary and two real with opposite signs. The appropriate one is real with positive sign.

### 3 Case study

As a demonstration of ZAC methodology we will compare the radar reflectivity of CWKR and CWSO radar sites in the CIS points for May 18, 2014 for a period of four hours, from 00:00 to 04:00 AM.

Figure 5 shows a scatter diagram for a raw data (clutter uncorrected) of radar reflectivity scanned at 2:30 AM. In the top frame points are colored according to the overlap coefficient value in the CIS point: yellow for overlapping coefficient  $\psi \leq 0.25$ , green for  $0.25 < \psi \leq 0.50$ , and blue for  $0.50 < \psi \leq 0.75$ . In the bottom frame of Figure 5 points are similarly coloured according to value of the time synchronization coefficient: blue for  $0.50 < \xi \leq 0.75$  and red for  $0.75 < \xi \leq 1$ . From the presented example, the correlation in CIS points is very good since the raw data were used. We did not apply the complete ZAC methodology since we did not have a whole set of overlap coefficients for the composite volume created from multiple sampling of 1 degree azimuth resolution. Instead, we used values of overlapping coefficients for overlap of individual beams (with beam width of  $0.62^\circ$ ).

As a comparison method, after each volume scan (10 minutes duration), the mean logarithmic radar reflectivity factor was calculated for a “box” of dimensions 6km x 20km x 120km in the middle between two radars perpendicular to the line from WKR to WSO. The difference between two reflectivity  $dZ_{\text{box}} = Z_{\text{WSO}} - Z_{\text{WKR}}$ , was plotted in time series of Figure 6 (red) together with the differences  $dZ_{\text{cis}}$  obtained from the regression line of Z in the CIS points (black).

In both methods of dZ calculation WSO radar had lower values than WKR. The “box” method gave twice the difference as the CIS method,  $dZ_{\text{box}} \sim -3\text{dB}$  versus  $dZ_{\text{cis}} \sim -1.5\text{ dB}$ . Also the range of dispersion of the “box” model was more than two times greater than the CIS method, 7 dB versus 3 dB.

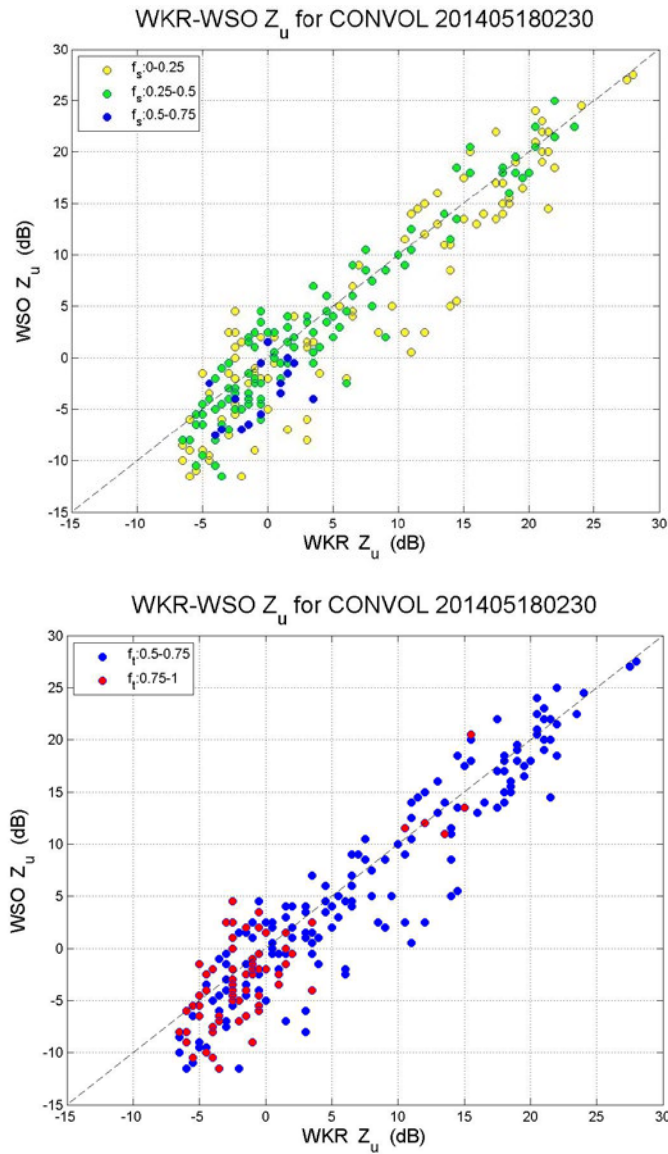


Figure 5: A scatter diagrams for clutter uncorrected radar reflectivity for WKR-WSO radar site for May 18, 2014 at 2:30 AM in the CIS points colored in regards to special overlap coefficients (top) and time synchronization coefficients (bottom).

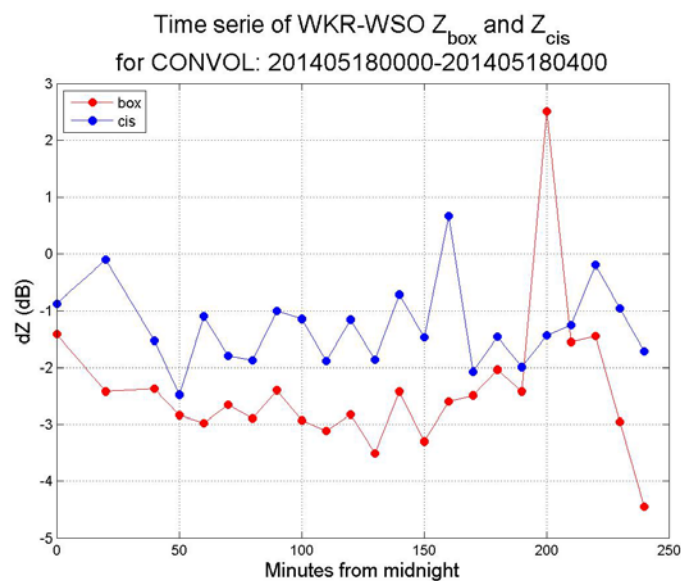


Figure 6: A time series of clutter uncorrected radar reflectivity differences between WKR and WSO radar sites for May 18, 2014 for time period of 4 hours (00:00-04:00 AM) calculated in the CIS points (blue) and as averaged value for a “box” dimension 6km x 20km x 120km.

#### 4 Limitations

So far we have not explicitly allowed for a number of real-world factors. The most obvious of these are attenuation of signals by heavy precipitation, since MSC uses mainly C-band data, and bending of the beam by atmospheric refraction. The first of these can be handled by data pre-selection that examines the paths to the CIS and removes pairs where there is significant reflectivity along the beam. Issues related to refraction are harder to handle. The radars we have examined so far have been located at similar heights and in similar environments, so errors in the absolute height should be approximately symmetric and the CIS remains aligned. For radars at differing heights the beam geometry can be adjusted to reflect a standard propagation model, such as the 4/3 model (Doviak, 2006). Nonetheless it seems prudent to assume that a reliable point-by-point inter-comparison methodology will need to pre-select against cases where significant anomalous propagation is likely.

#### 5 Conclusion

The core of the ZAC methodology is the ability to obtain accurate coordinates of a geometric set of points at an equal distance from both radars, the CIS points. Consequently, knowing the CIS points we can find matching local radar coordinates of the CIS point that will give a pair of measured radar reflectivity from both adjacent radars.

The ZAC methodology quantifies the lack of exact alignment of operational data in time and space. Weighting factors for spatial and time alignment of operational data were introduced and modeled as function of relative overlapping of radar beam volumes and nonsynchronization. As an illustration, the space and time factors were calculated for individual matching radar sample pulse, instead of the integration of all pulses (samples) during the previous resolution degree of antenna motion. To avoid a possible misinterpretation of numerical results, calculated weight factors were used only for the graphical indication of the quality of the CIS points without a full application of the ZAC methodology. Even with such reduced implementation of the ZAC methodology it has shown a great potential for future development and operational usage.

#### References

**Atlas, D:** Radar Calibration: Some Simple Approaches, Bulletin of the American Meteorological Society – 2002, Volume 83, Issue 9 (September 2002) pp. 1313-1316.

**Doviak, R. J. Doviak and Zrnic, D.S.:** Doppler Radar and Weather Observations. Second edition. Dover Publication - 2006.

**Ulbrich C. W., and Lee L. G.:** Rainfall Measurement Error by WSR-88D Radars due to Variations in  $Z-R$  Law Parameters and the Radar Constant, Journal of Atmospheric and Oceanic Technology – 1999, Volume 16, Issue 8 (August 1999) pp. 1017-1024.

**Vukovic, Z.R., Young, J.M. and Donaldson, N:** Determining Geometry for Common Comparison Radar Space, <https://ams.confex.com/ams/36Radar/webprogram/Paper228787.html>, 36th Conference on Radar Meteorology (16-20 September, 2013), Breckenridge, CO, US.

**Williams. C.R., Gage K.S., Clark, W. and, Kucera P.:** Monitoring the Reflectivity Calibration of a Scanning Radar Using a Profiling Radar and a Disdrometer, Journal of Atmospheric and Oceanic Technology - 2005, Volume 22, Issue 7 (July 2005) pp. 1004-1018.