Performance comparison of receiving filters/algorithms in weather radar

Andrés Martínez Mera[1] and Verónica Santalla Del Río[1]

[1] *Signal Theory and Communications Department, University of Vigo, Spain*
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Andrés Martínez Mera

1. Introduction

From an operational point of view, obtaining low variance and unbiased estimators is essential for a desirable overall performance in any weather radar. For this reason, much research has been done in order to improve the quality of these estimators which led to two different signal processing techniques.

In the early years of the weather radar research, the matched filter was generally implemented because of its simplicity and the interest in optimizing the SNR. Later, technological advances made possible to implement pulse compression techniques. A complete review of pulse compression techniques applied to weather radar was done in [Mudukutore and Chandrasekar (1998)]. Using pulse compression, smaller range cells were obtained leading to a sharp increase in the reliability of the weather forecasts and severe storm alarms at the expense of a larger signal bandwidth required. Most importantly, extremely low sidelobes and Doppler robustness were achieved what suggested that these techniques deserve further research. However, for example, Barker codes (in particular the long ones) need complex hardware at the same time they require wide bandwidth, which becomes a problem directly related with both the cost of the final product and the noise performance. Even though pulse compression was initially thought to improve range resolution, by considering range averaging, the quality of the power estimators in terms of bias and variance could be improved by widening the resolution cells.

The higher oversampling rates achieved by new analog to digital converters made possible to use whitening transformation based signal processing. Considering that the channel impulse response can be modelled as an uncorrelated process, the autocorrelation function of the received signal is directly obtained from the transmitted pulse autocorrelation. Data samples corresponding to each range gate are then separately whitened and then averaged. An important advantage of whitening as compared to pulse compression is that it exploits efficiently the spectrum since it does not require sophisticated pulse waveforms. Nonetheless, whitening transformations always carry within themshelves an important noise enhancement what becomes a major problem when it comes to low SNR.

To date, pure whitening (WTB) [Ivić et al. (2003)] and softer transformations such as the sharpening transformation (SFB) [Torres et al. (2004)] or the adaptative pseudowhitening method (APTB) [Curtis and Torres (2011)] have only been theoretically assessed under range stationary conditions whereas [Mudukutore and Chandrasekar (1998)] studies several inverse filters through non-stationary reflectivity profiles. It is important, then, to assess the behavior of whitening techniques in the case of non stationary channels in range.

All the same, in weather radar signal processing it exists a tight tradeoff between bandwidth, noise and range resolution what means that one of these parameters is necessarily worsen to improve another. Above all, these parameters must be carefully selected so as to meet accuracy constraints. With the aim of fairly assess and compare different processing schemes, this work evaluates a set of figures of merit which shows the strengths and weaknesses of the different techniques at reception.

2. Channel modeling and receiving schemes

For the purpose of the paper, the radar system can be modelled using the equivalent discrete-time channel. The notation below will be considered along the paper:

- r[n]: Signal at the receiver input.
- s[n]: Signal after filtering/whitening.
- p[n]: Transmitted pulse.
- C[n, t]: Time variant channel coefficients. It is assumed that the channel coefficients follow a circular complex Gaussian stationary process in the variable t and a non-stationary uncorrelated process in the range variable n.
- h[n]: Receiving filter (M coefficients).
- $ph[n] = p[n] * h[n]$
- W_T: Matrix for whitening transformation.

- N[n]: Additive noise. White noise is assumed.
- N_0: Index which denotes the beginning of an arbitrary range cell (L samples).
- $R_p[n]$: Autocorrelation function of the transmitted pulse, p[n].
- $R_h[n]$: Autocorrelation function of the reception filter, h[n].
- $W_L(1,\Sigma)$: Wishart distribution of $X^{*T}X$ where X is a row vector and follows a $N_L(0,\Sigma)$ distribution [Maiwald and Kraus (2000)]
- C_p: Correlation matrix of the transmitted pulse. [Ivić et al. (2003)]

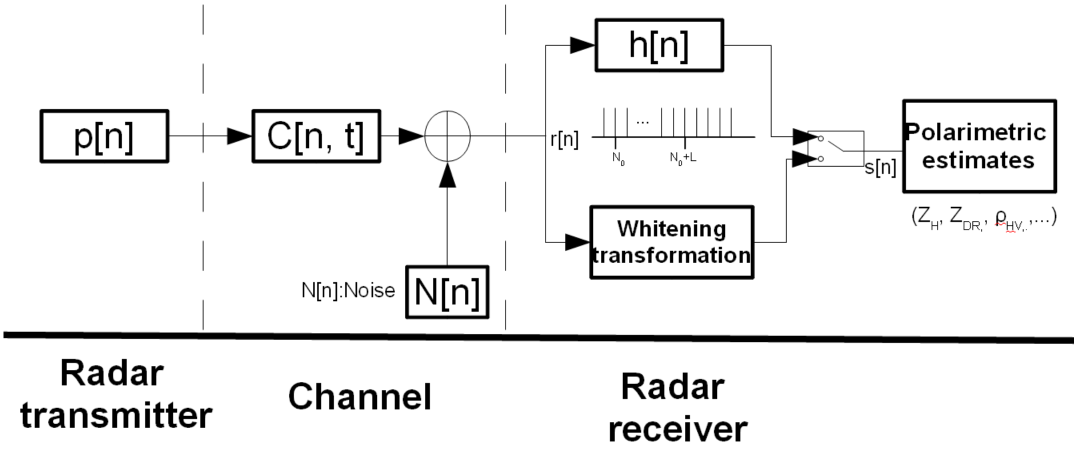

Figure 1: Radar model diagram

The received signal can be expressed as:

$$r[n] = p[n] * C[n] + N[n] \tag{2.1}$$

This signal will be either filtered or, windowed and whitened, to finally obtain the power estimate for each range cell. This will be obtained from $s[n]$ by non-coherent averaging of succesive power samples as:

$$\hat{P}_D = \frac{1}{L}\sum_{n=N_0}^{N_0+L-1} |s[n]|^2 - \hat{P}_N \tag{2.2}$$

where the power noise estimator, $\hat{P}_N$, is assumed to be obtained in the abscense of backscattered signal.

2.1. Filtering

In the case that a receiver filter is used, $s[n]$ is given by:

$$s[n] = r[n] * h[n] = p[n] * c[n] * h[n] + N[n] * h[n] = s_D[n] + s_N[n] \tag{2.3}$$

where $s_D[n]$ and $s_N[n]$ are the contributions from the backscattered rain signal and the noise respectively.

2.2. Whitening

The pure whitening approach proposed in [Ivić et al. (2003)], which laid the foundations to more advanced transformations, considers an uncorrelated stationary channel response in range so that the received signal can be modelled as a correlated stationary process whose autocorrelation function is determined by the transmitted pulse, $R_p[n]$.

To obtain the power estimate for each range cell, L samples of $r[n]$ corresponding to that range gate, are taken and arranged into a column vector, $\bar{r}$, which is multiplied by the whitening matrix W_T. That is:

$$\bar{s} = W_T \cdot \bar{r} = W_T \cdot \bar{r}_D + W_T \cdot \bar{r}_N = \bar{s}_D + \bar{s}_N \tag{2.4}$$

where $\bar{s}_D$ and $\bar{s}_N$ are the contributions from the rain backscattered signal and from the receiver noise. The whitening matrix, W_T, can be calculated according to WTB, SFB or APTB algorithms. In all cases, it was assumed that the correlation matrix of the received samples depends only on the autocorrelation of the transmitted pulse.

3. Figures of merit

3.1. Variance reduction, equivalent number of independent samples

The main purpose of the whitening techniques was to reduce the variance of the received signal by taking advantage of the higher number of samples available from a range cell. To assess the quality of the whitening transformation, the equivalent number of independent samples was used [Ivić et al. (2003)]. That is, the number of samples from the rain cell, that, if they were independent, they would give the same variance (in the abscence of noise) as L whitened samples. Evidently, the equivalent number of independent samples will range from 1 to L, being L the desired result.

3.2. Noise figure

Either if filtering or whitening are considered, the noise is increased. This implies a decrease of the signal to noise ratio (SNR) between the input and the output of the receiver. The noise figure (NF), defined as the ratio of SNR between input and output ports, gives a measure of the noise gain.

For the filtering case, using a stationary rain channel as a reference situation, it has been found to be

$$NF = \frac{SNR_{in}}{SNR_{out}} = \frac{R_p[0]R_h[0]}{(R_p * R_h)[0]} \tag{3.1}$$

In the case of whitening it can be calculated as [Ivić et al. (2003)]

$$NF = \frac{tr(W_T^* W_T^T)}{tr(W_T^* C_p W_T^T)} \tag{3.2}$$

3.3. Power bias

The power estimator considered is unbiased as long as the rain channel is stationary in range. If this is not the case, as usually happens, this estimator becomes biased.

This bias can be calculated as the difference between the expected value of the estimator in a range cell and the true value of power corresponding to that range cell. This is a consequence of the power leakage towards adjacent cells that convolutional processes imply. Obviously, the bias depends on the specific rain profile, so a specific profile has been considered to evaluate it. In particular, it has been considered that rain is present in just one range cell (L samples), and within that cell is homogeneous, that is, all samples have equal power

For this rain profile, when filtering is considered in reception, the bias can be obtained as:

$$Bias = E\{\hat{P}\} - P_c \tag{3.3}$$

where P_c is the power from one sample of the rain cell. Considering the backscattering from rain and the noise are independent,

$$E\{\hat{P}\} = E\{\hat{P}_D\} = \frac{tr(\Sigma \cdot PH)}{L} \tag{3.4}$$

where $\Sigma = P_c \cdot I_L$ is the covariance matrix of the rain cell samples, I_L is the identity matrix and

$$PH(k,m) = \sum_{n=N_0}^{N_0+L-1} ph[n-k]ph^*[n-m], PH \in \mathbb{C}^{L \times L} \tag{3.5}$$

In the case of whitening, the bias results to be:

$$Bias = E\{\hat{P}\} - P_c = E\{\hat{P}_D\} - P_c = \frac{1}{L} tr(W_T^{*T} \cdot W_T \cdot E\{\bar{r}_D \cdot \bar{r}_D^{*T}\}) - P_c \tag{3.6}$$

3.4. PSL and ISL

These parameters have been defined in [Mudukutore and Chandrasekar (1998)] to quantify the power leakage towards adjacent range cells:

$$PSL = 10log_{10}\left(\frac{\text{Peak sidelobe level}}{\text{Total mainlobe power}}\right) \tag{3.7}$$

$$ISL = 10log_{10}\left(\frac{\text{Power integrated over sidelobes}}{\text{Total mainlobe power}}\right) \tag{3.8}$$

Clearly, the higher PSL/ISL, the better accuracy. The PSL indicates the capability of the system to detect reflectivity gradients whereas the ISL is closely related to the bias of the power estimator.

4. Results

The following results show the figures of merit of different receiving filters and whitening transfomations for different oversampling rates. For the sake of clarity, the following notation is used:

- RP: Rectangular pulse, every element equal to one.
- P4: P4 coded pulse [Levanon (2004)]
- $L \times k$: Length of the receiving filter. $k \in \mathbb{N}$
- MF: Matched Filter
- IF: Inverse Filter

A reflectivity gradient sweep from 0 dB to 40 dB is done in order to assess the bias of the power estimator. As for whitening transformations, WTB and SFB are considered.

4.1. Bias

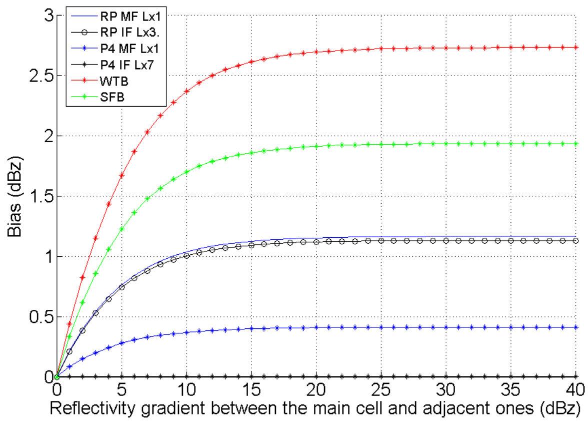

Figure 2: Power bias versus reflectivity gradient in adjacent cells (L=5).

Figure (2) shows that regarding WTB, the power estimator is unbiased as long as the channel is stationary. However, for non-stationary channels it is biased 2.7 dB for gradients bigger than 15dB. SFB performs slightly better.

4.2. Noise Figure

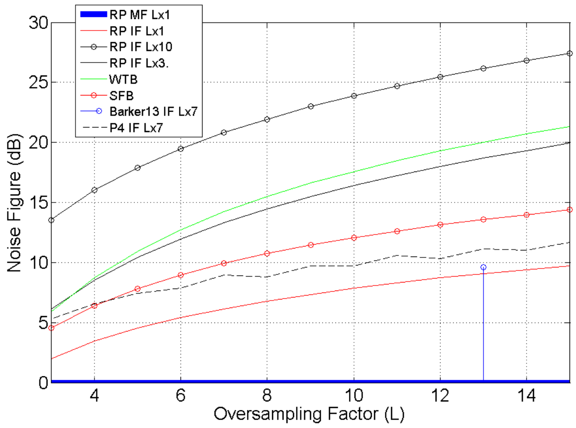

Figure 3: Noise Figure

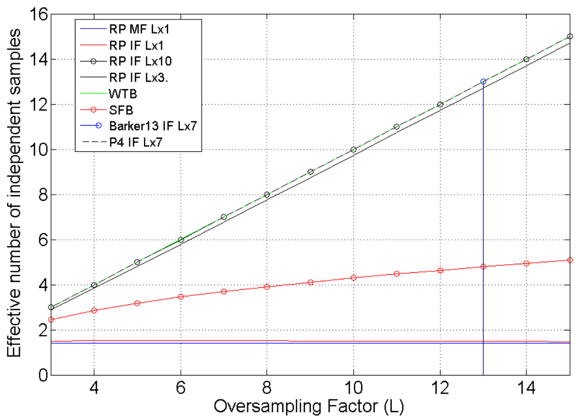

Figure 4: Equivalent number of independent samples

Fig. (4) reveals that good decorrelation is achieved for both inverse filtering and whitening. However, variance reduction worsens considerably as soon as the whitening transformation differs from pure whitening. As an example, SFB was designed as a low noise whitening transformation but, when it comes to accuracy it cannot reach variance reduction ratios such as WTB. Regarding MMSE inverse filters, it cannot be argued that Barker and P4 codes are capable of bringing quite good decorrelation and reasonable noise levels Fig. (3), although the bandwidth requirements rise dramatically.

4.3. PSL

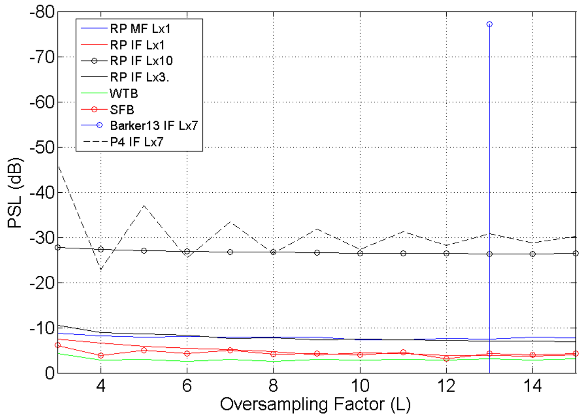

Figure 5: PSL

4.4. ISL

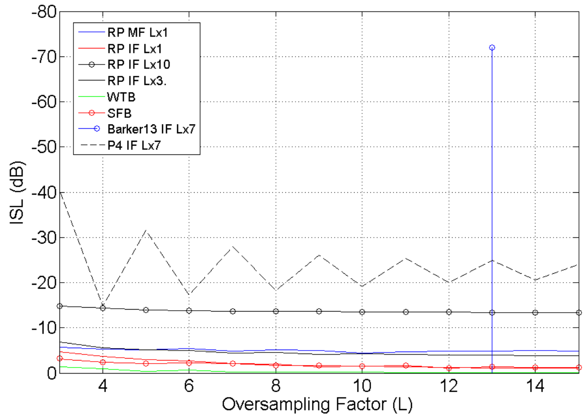

Figure 6: ISL

Certain weather phenomena may produce strong gradients of reflectivity (30-40 $dBkm^{-1}$) [Bringi and Chandrasekar (2001)] what means that PSL must be low enough ($\sim$ 40 dB) so as to avoid masking. Bearing in mind the Figure (5), most receiving schemes cannot meet this requirement. Specifically whitening transformations, WTB and SFB exhibit poor PSL results what is serious drawback. Regarding ISL, whitening transformations as well as the matched filter yield high ISL levels what implies that bias may not be neglected.

5. Conclusions

At the light of the results and the theoretical foundations, whitening processing does not seem to be an appropiate solution for unknown weather situations since the whitening transformation is designed not to fit an arbitrary reflectivity profile but an uniform one. Unlike filters, noise enhancement in whitening varies depending on the covariance matrix of the samples (i.e. on the reflectivity profile) what means that different range cells are tied to different noise levels.

On the other hand, phase coded sequences such as P4 and Barker tend to achieve both unbiased and low variance results. Besides this, noise gain is reasonable and low PSL/ISL is achieved. Nevertheless, these codes cannot meet tight bandwidth constraints since they are wideband signals.

Finally, results evidence that to date it does not exist a receiving scheme capable of meeting bandwidth, accuracy, power and noise constraints, so it seems that the signal design problem is still open and the research for alternative algorithms should continue.

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