Optimization of turbulence measurements for radar, lidar and sonic anemometers

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1. Introduction

Aviation has an increasing interest in monitoring the local wind field to enhance aviation safety and to increase the airport capacity. To this end measurements of turbulence intensity, wake vortex detection and wake vortex monitoring are important. The turbulence intensity can be quantified with the eddy dissipation rate (EDR). At this time no generally accepted algorithm for EDR retrievals from experimental data exists. An EDR retrieval algorithm intended for turbulence warning and forecasting must be able to resolve 1. high-intensity values to avoid turbulence encounters and 2. low-intensity values that enhances the wake vortex lifetime. Comparisons of EDR retrievals by different instruments are made in several studies, e.g. Chan (2011); Meischner et al. (2001); O'Connor et al. (2010). EDR can be retrieved from measurements of in-situ sonic anemometers; remotely from radar / lidar observations; with balloon borne soundings or with aircraft measurements. Spatial or temporal sampling of the signal may be used, where the signal can be velocity, humidity or temperature. Further analysis gives the EDR, by using the variance, structure functions or a least squares fit of the power spectrum. An essential problem with all these measurements is that there is no reference to true EDR...

Turbulence, is hypothesized by Kolmogorov as a special solution of the Navier-Stokes equations, with assumptions on homogeneity and isotropy and satisfying the transfer of energy from larger to smaller scales (Pope, 2000). The turbulence intensity is quantified by the eddy dissipation rate (EDR), and it is statistically represented by the turbulent energy spectrum of the velocities, known as the Kolmogorov -5/3 power law:

$$E(\kappa) = \quad C\epsilon^{2/3}\kappa^{-5/3}, \tag{1.1}$$

where $\kappa = 2\pi/l$, is the wavenumber, with κ in the inertial range and l being the length scale, C, is the universal Kolmogorov constant, and ϵ is the eddy dissipation rate. For the longitudinal and transverse turbulent energy spectrum an adjusted constant is used. Further we have the second order structure function, which can also be used for retrievals:

$$D_{LL}(r) =< [v(x+r) - v(x)]^2 >= C_2(\epsilon r)^{2/3} \tag{1.2}$$

where C_2 is a universal constant for the longitudinal direction. For the appropriate constants we refer to Pope (2000).

In this study we estimate optimal settings, by applying retrievals to synthetic observations and estimate the bias and precision for radar, lidar and sonic anemometers. A turbulence wind field is created by using the cascade turbulence model, see Fig. 1. Consequently measurements are generated with as input the characteristics of the measurement device, that is either the sonic anemometer, the lidar or the radar. These characteristics include e.g. the beam width, range resolution, cloud structure, etc. The EDR retrievals are applied to see whether they are biased and what the precision of the retrieval algorithm and instrument is. The optimal settings consider the number of measurements, threshold for a minimal detectable eddy dissipation rate and the resulting error characteristics of the retrieved EDR. Consequently the retrievals are applied to measurements from the Cabauw research site in the Netherlands to see whether the predicted biases between different instruments coincides with the predictions from the synthetic experiments.

In the following of this extended abstract a simple example is provided to introduce the reader to the topic. In this example the variance method to obtain EDR from the sonic anemometers is considered by applying error analysis and performing a synthetic experiment. The number of samples to estimate the variance is varied, and the inertial range approximation is checked.

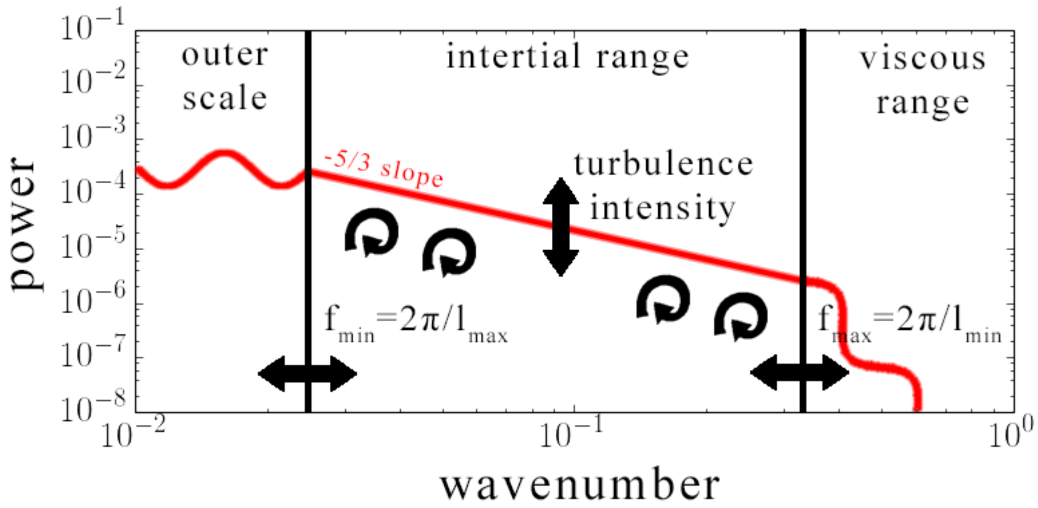


*Figure 1: **Sketch of the atmospheric cascade turbulence model**. Atmospheric turbulence is simulated by using a cascade turbulence model: 21 samples are used for a turbulence realization on an infinite space domain $[-\infty, \infty]$, on a predefined frequency domain [f_{min}, f_{max}]. The simulated velocities are periodic on each scale, and self-similar on each scale. The turbulent velocities satisfies the Kolmogorov -5/3 power law and the Kolmogorov second order structure function. For more details on the cascade turbulence model, see Oude Nijhuis et al. (2014).*

2. Example: EDR from the velocity variance of sonic anemometers

In this example we derive an equation that relates EDR to the variance of a velocity series from a device that has a fixed location. We apply error analysis to this equation to derive the contribution from different error sources. After that we simulate an experiment with the cascade turbulence model. The results give us an idea what we can expect from EDR measurements.

First we note that the power spectrum, that applies to the spatial domain, can be written for the time domain as:

$$E'(\chi) = C\epsilon^{2/3}\chi^{-5/3}U_0^{2/3}, \tag{2.1}$$

where χ is the frequency and U_0 the average wind speed, with which the turbulence is advected. Given the variance of a velocity series, from e.g. a sonic anemometer, we can relate the variance to EDR via integration of the power spectrum (Bouniol et al., 2003; O'Connor et al., 2010; Shupe et al., 2012):

$$\sigma_T^2 \quad = \int_{\chi_1}^{\chi_2} E'(\chi)d\chi = \tfrac{3}{2}C\epsilon^{2/3}U_0^{2/3}\left[\chi_1^{-2/3} - \chi_2^{-2/3}\right] \tag{2.2}$$

where σ_T^2 is the variance in the temporal domain and $\chi_{1,2} = 2\pi U_0/l_{1,2}$ the frequencies of integration, and U_0 the average wind speed. The eddy dissipation rate can then be found by solving for ϵ:

$$\epsilon = \left(\frac{3}{2}C\left[\chi_1^{-2/3} - \chi_2^{-2/3}\right]\right)^{-3/2} U_0^{-1}\sigma_T^3 \tag{2.3}$$

Error analysis can be applied to the eddy dissipation rate ϵ, assuming uncorrelated and Gaussian distributed errors. See e.g. Taylor (1997). As ϵ varies on different orders of magnitude it is natural to prefer a different variable. Here we use $\epsilon^{1/3}$ to apply the error analysis on because $\epsilon^{1/3}$ scales with the standard deviation of the velocities σ_T. We can write:

$$(\sigma_{\epsilon^{1/3}})^2 = \sum_i \left(\frac{\partial\epsilon^{1/3}}{\partial\epsilon}\frac{\partial\epsilon}{\partial\alpha_i}\sigma_{\alpha,i}\right)^2, \tag{2.4}$$

where the summation is over all the relevant variables α_i. This is expression is written more convenient as:

$$\left(\frac{3\sigma_{\epsilon^{1/3}}}{\epsilon^{1/3}}\right)^2 = \sum_i \left(\frac{1}{\epsilon}\frac{\partial\epsilon}{\partial\alpha_i}\sigma_{\alpha,i}\right)^2. \tag{2.5}$$

Working out the error analysis for the time domain we obtain:

$$\left(\frac{3\sigma_{\epsilon^{1/3}}}{\epsilon^{1/3}}\right)^2 = \left(\frac{1}{\epsilon}\frac{\partial\epsilon}{\partial\chi_2}\sigma_{\chi,2}\right)^2 + \left(\frac{1}{\epsilon}\frac{\partial\epsilon}{\partial\chi_1}\sigma_{\chi,1}\right)^2 + \left(\frac{1}{\epsilon}\frac{\partial\epsilon}{\partial U_0}\sigma_{U,0}\right)^2 + \left(\frac{1}{\epsilon}\frac{\partial\epsilon}{\partial\sigma_T}\sigma_{\sigma,T}\right)^2, \tag{2.6}$$

$$\left(\frac{3\sigma_{\epsilon^{1/3}}}{\epsilon^{1/3}}\right)^2 = \frac{\left[\chi_2^{-10/3}\sigma_{\chi,2}^2 + \chi_1^{-10/3}\sigma_{\chi,1}^2\right]}{\left[\chi_2^{-2/3} - \chi_1^{-2/3}\right]^2} + \left(\frac{\sigma_{U,0}}{U_0}\right)^2 + \frac{9}{2(N-1)}, \tag{2.7}$$

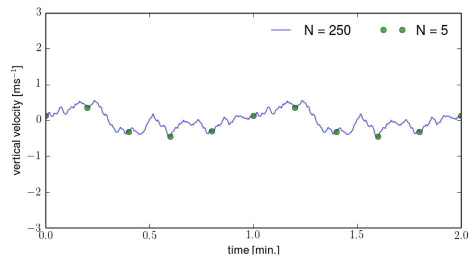
$$\left(\frac{3\sigma_{\epsilon^{1/3}}}{\epsilon^{1/3}}\right)^2 \approx 1 + \left(\frac{\sigma_{U,0}}{U_0}\right)^2 + \frac{9}{2(N-1)} \text{ if } \chi_1 \ll \chi_2 \text{ and } \sigma_{\chi,1} \approx \chi_1. \tag{2.8}$$

where N is the number of samples. Here we used that the standard deviation of the standard deviation $\sigma_{\sigma,s}/\sigma_s = [2(N-1)]^{-1/2}$ (Taylor, 1997). Further we assumed that the error in the frequency is $\sigma_{\chi,1} \approx \chi_1$. From this analysis, Eq. 2.8, we can see that:

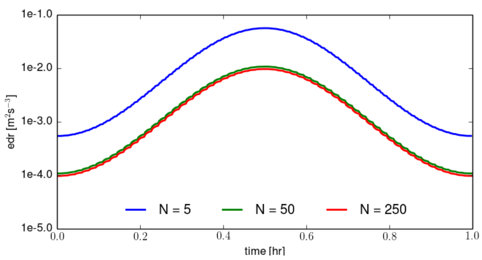
- If the number of samples is sufficient, more than 50, then undersampling is not causing the major error because $\frac{9}{2(N-1)} < 0.1$.
- The variance in average velocity, $\sigma_{U,0}$, either due to the outer scale motions, horizontal shear or eddies themselves may have a large impact on the error in EDR.
- This error analysis may be limited because the error in the minimal frequency χ_1 may be much larger than $\sigma_{\chi,1}$.

2.1. Experiment with synthetic data

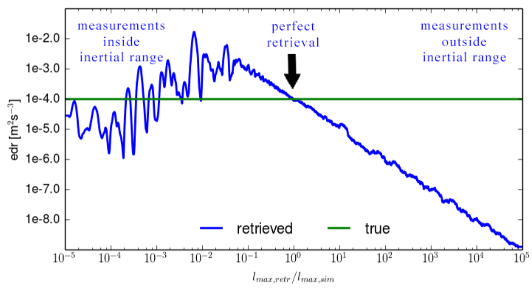
In Fig. 2 the cascade turbulence model is used to simulate a velocity signal with a prescribed ϵ, eddy dissipation rate, and $l_{max,sim}$, the maximum length scale of the inertial range. We can see in Fig. 2(b) that a low number of samples, e.g. $N = 5$, can induces a bias on the retrieved EDR. If the number of samples is large enough, $N > 50$, this is not a problem anymore. A problem with this EDR retrieval is that $l_{max,sim}$, the maximum length scale of the inertial range is unknown. This is shown in Fig. 2(c). The measurements can be outside the inertial range, causing a bias. Or the measurements can be inside the inertial range where the eddies outside the maximum observed length scale can still have an impact on the retrieved EDR. Here we estimate that when the measurements are in the inertial range, we can expect a bias of up to an order of magnitude. this can cause a bias up to several order of magnitude due to the unknown maximum length scale of the inertial range.



(a) vertical wind velocity



(b) different number of samples N



(c) mismatch of l_{max} (or f_{min})

*Figure 2: **Simulation of EDR retrieval from the velocity variance** The cascade turbulence model is used to simulate EDR retrievals from the velocity variance. This simulations resembles measurements from the sonic anemometer. In the retrieval N samples are used. (a) vertical wind velocities for short period; (b) Retrieved EDR with different number of samples, N; (c) Mismatch of $l_{max,retr}$ in the retrieval compared to $l_{max,sim}$ the simulation. If $l_{max,retr} > l_{max,sim}$ the measurements are performed outside the inertial range. If $l_{max,sim} < l_{max,retr}$ the measurements are inside the inertial range, however the larger eddies can still have an influence.*

3. Outlook

Here we have given an example on how an EDR retrieval can be analyzed. In the presentation focus will be put on EDR from radar measurements. The EDR retrievals are applied to measurements from the Cabauw research site in the Netherlands to see whether the predicted biases between different instruments coincides with the predictions from the synthetic experiments.

Acknowledgement

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