# Comparison of measured and radar-equivalent gamma DSDs to investigate the effect of gamma raindrop size distribution assumption on the rain rates

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## 1 Introduction

The knowledge of rain drop size distribution (DSD) has a wide range of applications in earth sciences such as precipitation physics, hydrology, and agricultural and soil sciences. DSD is notoriously important in developing retrieval algorithms in precipitation remote sensing. In radar meteorology all the radar rainfall algorithms are, at different extent, sensitive to DSD variability. Although literature reports different parametric forms to model the measured DSD, to date, the commonly most used distribution is the three-parameter gamma (Ulbrich, 1983). The goal of this study is to evaluate the error, in terms of rain rate, due to this assumption. A methodology was set up to compare the rain rates of a disdrometer-measured DSD and a simulated gamma DSD equivalent in terms of radar measurements (reflectivity factor, $Z_h$, differential reflectivity, $Z_{dr}$, and specific differential phase shift, $K_{dp}$). The differences, expressed in terms of normalized standard error (NSE) and normalized bias (NB) computed between the two rain rates will provide an indication about the influence of the gamma assumption on rain rate estimation. The influence of other factors, such as the raindrop shape model, the integration time interval, the radar frequency, and the influence of disdrometer sampling error are also investigated.

## 2 Methodology

Given a DSD (simulated or measured) the dual-polarization radar measurements can be estimated using electromagnetic models at the typical weather radar frequencies, while the rain rate is computed as follow:

$$R = 6\,\pi\,10^{-4} \int_0^{\infty} v(D) N(D) D^3\, dD \qquad (mm\ h^{-1}) \tag{2.1}$$

where *v(D)* is the drop terminal fall speed (Gunn and Kinzer, 1949). The criterion to match radar triplets computed from measured spectra with triplets computed from a widely variable set of gamma DSDs is obtained by combining two procedures: the first uses a cost function to identify the nearest neighbour of radar measurement triplet in the simulated dataset, while the second one uses a 3D interpolating function. A cost function describes the distance between two points in the 3D space defined by the three radar measurements and is

$$CF = \frac{\left(Z_{h,m} - Z_{h,s}\right)^2}{\overline{Z_{h,s}}} + \frac{\left(Z_{dr,m} - Z_{dr,s}\right)^2}{\overline{Z_{dr,s}}} + \frac{\left(K_{dp,m} - K_{dp,s}\right)^2}{\overline{K_{dp,s}}} \tag{2.2}$$

where the subscripts *m* and *s* indicate that measurements were obtained from measured and simulated DSDs, respectively. The gamma DSD that minimizes (2.2) is the one that matches the given measured DSD. To obtain a meaningful match, the maximum value for the cost function was defined as 0.001. Then the values of rain rates obtained from the gamma simulated DSDs located in the 3D space around the given measured DSD are investigated. If not homogeneous (difference between maximum and minimum value greater than 15 mm $h^{-1}$) the matching is considered unreliable, since a small change in the values of the radar triplet can cause a relatively large change in the value of the rain rate. The second procedure consists in defining a function that fits a surface of the form $R_s = f(Z_{h,s}, Z_{dr,s}, K_{dp,s})$ to the scatter data $Z_{h,s}$, $Z_{dr,s}$, $K_{dp,s}$, and $R_s$. Knowing the two values of the simulated rain rate (one obtained from the cost function and one from the interpolating function) a quality control that permits to avoid erroneous result obtained particularly on the border of the 3D space of the radar measurements is applied. If the difference of rain rates obtained from the two methods is greater than 4 mm $h^{-1}$ the measured DSD is discarded. The percentages of discarded DSD ranges between 2% and 7% and the criterion based on the cost function threshold ($CF < 0.001$) plays the most important role.

## 3 Experimental and Simulated Datasets

Observational data used in the present study consist of 1-min spectra collected by two-dimensional video disdrometer (2DVD) in four different sites, namely *i)* at the National Space Science Technology Center (NSSTC) in Huntsville, Alabama, *ii)* at Emäsalo, Finland, during the Light Precipitation Evaluation Experiment (LPVEx), *iii)* in Central Oklahoma during the Midlatitude Continental Convective Clouds Experiment (MC3E), and *iv)* at Rome, Italy, during the Special Observing Period 1 (SOP1) of the Hydrological Cycle in Mediterranean Experiment (HyMeX). MC3E data were collected by five different 2DVDs. The 2DVD is an optical device that provides the estimates of several observables of a drop, namely

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equivolume diameter (*D* in mm), the volume, the fall speed, the axis ratio, and the cross-sectional area of each drop that falls in the virtual measuring area of 10×10 $cm^2$ (Schönhuber et al., 2007). From these measurements the empirical DSD was obtained by stratified observed diameters *D* into 50 bins with a constant width of 0.2 mm. Spurious drops due to splashing or wind effect were removed using the filter criterion of Tokay et al. (2001), while a threshold of 10 drops and rain rate greater than 0.01 mm $h^{-1}$ are used to identify a rainy minute.

The simulated dataset was built by randomly varying the three parameters of the normalized gamma DSD, namely $N_w$, $D_0$, and μ within some specified intervals. A frequently used set of intervals is $0.5 \leq D_0 \leq 3.5$ mm, $-1 \leq \mu \leq 5$, and $10^3 \leq N_w \leq 10^5$ $mm^{-1}$ $m^{-3}$ with the additional constraints of rain rates lower than 300 mm $h^{-1}$ and ($10 \log_{10} Z_h$) < 55 dBZ at S-band (Bringi and Cheandrasekar, 2001; Gorgucci et al., 2008). In this study wider intervals are used: a sensitivity study allowed to decide to adopt the following intervals: $0.5 \leq D_0 \leq 3.5$ mm, $-4 \leq \mu \leq 20$ and, $10^1 \leq N_w \leq 10^7$ $mm^{-1}$ $m^{-3}$. The gamma simulated dataset consists of more than 80000 gamma DSDs. It should be noted that for the purpose of this study is not important to identify a "realistic" gamma DSD parameter intervals, but an appropriately wide range of gamma DSD parameters to account for the variability of the radar measurement triplets determined from the disdrometer measured DSDs.

## 4 Results

The matching methodology exposed above will have an intrinsic error. It is evaluated in terms of rain rate once it is applied to a widely varying gamma simulated DSDs using a jackknife approach. Results show that such intrinsic error depends on the radar frequency: in terms of normalized standard error, 5%, 9%, and 14% were found for the S-, C-, and X-band, respectively, while the normalized bias (NB) is negligible for the three bands.

The influence of the disdrometer sampling error is estimated by applying the proposed matching methodology to a gamma DSD dataset resulting after simulating disdrometer sampling (Moisseev and Chandrasekar, 2007 and reference therein). The obtained NSE and NB depend now both on the intrinsic error due to the method and on the disdrometer sampling error. The NSE increases up to 9% for the S-band, 14% for the C-band and 20% for the X-band.

Finally, the proposed method was applied to the four measured datasets. To evaluate properties such as the median and the confidence interval of the NB and NSE, the bootstrapping method was adopted. Fig. 1 shows the boxplots of the resulting values of the NSE (Fig. 1a, b, c, and d) and NB (Fig. 1e, f, g, and h) obtained considering the following assumptions: S-band, shape model of Beard and Chuang (1987) and DSD integration time interval ranging from 1 to 10 minutes. Considering the four datasets, the values of the NSE range between 22% and 32% except for a few cases, while the values of the NB are never greater than 10%. The results are similar for all the datasets, although they were collected in four different climatic areas of the word. The dispersion of the NSE values is low and the trends of the medians are roughly smooth, indicating a very low influence of the integration time interval on the computation of rain rate using the gamma DSD assumption. Almost all the NB values are negative, indicating that the use of the gamma distribution to model the measured drop size spectra underestimates the rain rates. The NB values have a weak dispersion and a well defined trend, their absolute values slightly increasing with increasing integration time.

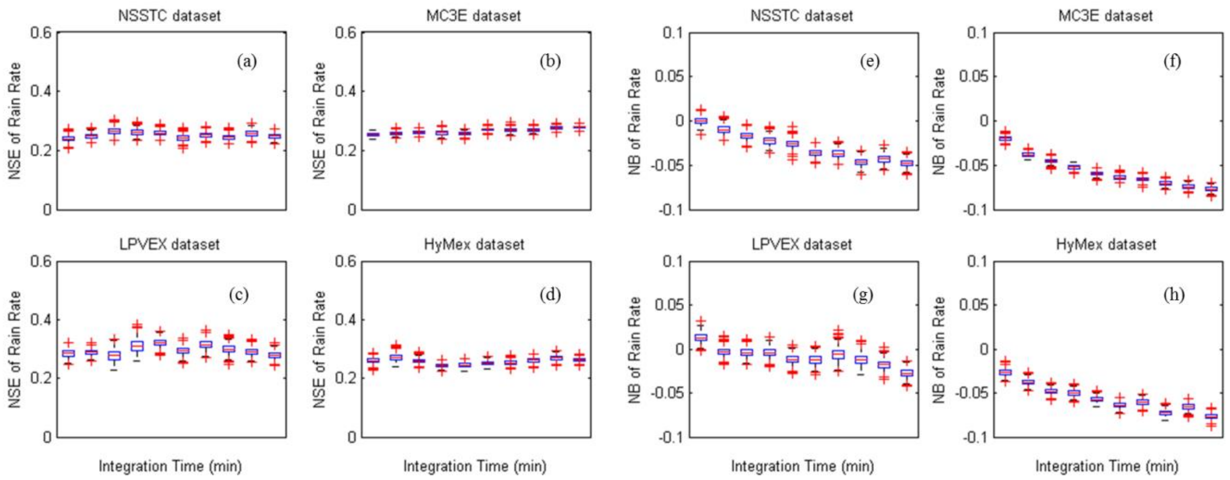


*Figure 1: Boxplots of the values of the NSE (a, b, c, and d) and NB (e, f, g, and h) between measured and simulated rain rate at the S-band for the shape model of Beard and Chuang (1987) and varying the integration time.*

Modelling the drop-shape with different shape size relations influence polarimetric radar measurements. The methodology was therefore applied to datasets built using different shape models, such as the one proposed by Pruppacher and Beard (1970), Brandes et al. (2002), and Gorgucci et al. 2000. The results (not shown) are in agreement with the ones obtained using the Beard and Chuang (1987) shape model. However, it seems that assuming a linear shape-size model, the assumption of a gamma DSD for describing natural drop size distribution has a more limited impact.

Finally the influence of the wavelengths has been investigated on disdrometer measured data. The radar measurements were computed also for the C- and X-bands using the same assumptions used for S-band, including shape-size relations and different integration time. The matching methodology has been then applied to the four disdrometer datasets. The results (not shown) underlined that the effect of the different wave lengths is not evident on the NB values: in fact, as for the S-band, the NB are lower than 10%, in most of the cases negative, and the absolute values increase as the integration time was increased. While the median values of the NSE are a few percentage points greater for the C- and X-bands than for the S-band (ranging around 30% and 33%, respectively, while for the S-band the NSE values range around 26%), and all the trends are similar. The dispersion of the NSE values is greater for the C-band and this effect can be attributed to the fact that at the C-band the resonance scattering is more significant compared to the S-band.

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## References

**Beard, K. V., and C. Chuang**, 1987: A new model for the equilibrium shape of raindrops, J. Atmos. Sci., 44, 1509-1524

**Brandes, E. A., G. Zhang and L. Vivekanandan**, 2002: Experiments in rainfall estimation with a polarimetric radar in a subtropical environment, J. Appl. Meteor., 41, 674-685

**Bringi, V. N., and V. Chandrasekar**, 2001: Polarimetric Doppler Weather Radar Principles and Applications. Cambridge University Press, 648 pp.

**Gorgucci, E., V. Chandrasekar, and L. Baldini**, 2008: Microphysical Retrievals from Dual-Polarization Radar Measurements at X-band, J. Atmos. Oceanic Technol., 25, 729-741

**Gorgucci, E., G. Scarchilli, V. Chandrasekar, and V. N. Bringi**, 2000: Measurement of mean raindrop shape from polarimetric radar observations, J. Atmos. Sci., 57, 3406-3413

**Gunn, R., and G. D. Kinzer**, 1949: The terminal velocity of fall for water droplets in stagnant air, J. Meteor., 6, 243-248

**Moisseev, D., and V. Chandrasekar**, 2007: Nonparametric estimation of raindrop size distributions from dual-polarization radar spectral observations, J. Atmos. Oceanic. Technol., 24,1008–1018

**Pruppacher, H. R., and K. V. Beard**, 1970: A wind investigation of the internal circulation and shape of water drops falling at terminal velocity in air, Quart. J. Roy. Meteor. Soc., 96, 247-256

**Schönhuber, M., G. Lammer, and W. L. Randeu**, 2007: One decade of imaging precipitation measurement by 2D-video-disdrometer, Adv. Geosci., 10 , 85-90

**Tokay, A., A. Kruger, and W. Krajewski**, 2001: Comparison of drop-size distribution measurements by impact and optical disdrometers, J. Appl. Meteor., 40, 2083-2097

**Ulbrich, C.**, 1983: Natural variations in the analytical form of the raindrop-size distribution, J. Climate Appl. Meteor., 22, 1764-1775